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I assumed that these expressions (in which the variables of integration were x and y, and the field of integration the radiating source) became identical [see (1) above] with the corresponding ones [(8) and (30)]* used by Rayleigh (in which the variables of integration are ξ, η, and the field of integration the focal plane). This would be the case when the limits of integration are infinite, if (as at first appears) the variables x, y were *symmetrically* involved with the variables ξ, η, in the expression for $I^2{}_1$. An inspection of (14) (of my paper), which expresses the value of r (the variable in I^2) in terms of x, y, ξ, η, shows that this is not the case. The first two variables are each multiplied by a factor of dissymmetry, $\frac{f}{D}$, and in order to obtain two new variables symmetrically involved with ξ, η, we must substitute

$$x = \frac{D}{f} d\xi_1, \qquad y = \frac{D}{f} d\eta_1,$$

which gives us in (19) and (31) respectively,

$$\int_{-\infty}^{+\infty} I^2 dy = \frac{D}{f}\int_{-\infty}^{+\infty} I^2 d\eta_1 = \frac{D}{f}(z_{,,}) \quad . \quad . \quad . \quad . \quad . \quad . \quad (19a)$$

and

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} I^2 dx dy = \frac{D^2}{f^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} I^2 d\xi_1 d\eta_1 = \frac{D^2}{f^2}(z_{,,,}) \quad . \quad . \quad . \quad (31a)$$

and the integrals [$(z_{,,})$ and $(z_{,,,})$] of *these last expressions* become respectively identical with the integrals (8) and (30) of Lord Rayleigh.

Yerkes Observatory: 1897 *December* 24.

The Concave Grating for Stellar Photography. By Charles Lane Poor and S. Alfred Mitchell.

(*Communicated by Dr. C. L. Poor.*)

The concave grating has proved a most powerful instrument for spectroscopic research, but heretofore it has not been successfully applied to stellar spectroscopy. Experiments are now being carried out at the Johns Hopkins University, under the direction of Dr. Charles Lane Poor, with the view of thoroughly testing the various methods of using the concave grating for astronomical purposes. The methods, originally suggested by Professor Rowland, were developed and the formulas derived by Dr. Poor, and the apparatus constructed under his direction; the photographs were made by Mr. S. Alfred Mitchell.

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* "Wave Theory," *Enc. Brit.* §§ 11 and 12.

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There are two radically different methods of using the concave grating for stellar work :

First, in connection with an objective ; the grating merely replacing the ordinary stellar spectroscope. This was tried by Professor Crew at the Lick Observatory in 1892 and a few results obtained.

Second, direct ; the grating is the objective and the spectroscope combined : the light from the star being reflected directly from the grating to the photographic plate. This method was first developed by Dr. Poor in 1892, when a few experiments were made. No definite results were then obtained, however. Experiments were resumed in October last, and some promising photographs have since been obtained.

This paper is confined to an explanation of the second method, to the derivation of the necessary formulas, and to a few notes in regard to the photographs already taken.

From the theory of the concave grating we have the general equation,

$$r = \frac{\mathrm{R}\,\rho \cos^2 \mu}{\mathrm{R}\,(\cos\mu + \cos\nu) - \rho\cos^2\nu} \qquad (1)$$

(see Rowland, *American Journal of Science*, vol. xxvi. 1883 Aug.). In this equation ρ is the radius of curvature of the grating, and the axis of the grating is the line of reference for angular measurements. R and ν are the spherical coordinates of the source of light ; r and μ those of the curve on which the spectra are brought to a focus.

This equation may be put into the following form :

$$r = \frac{\rho\cos^2\mu}{\cos\mu + \cos\nu - \dfrac{\rho}{\mathrm{R}}\cos^2\nu} \qquad (2)$$

If now the source of light is at an infinite distance, R is infinite, and the equation reduces to

$$r = \frac{\rho\cos^2\mu}{\cos\mu + \cos\nu} \qquad (3)$$

This is the general equation for the case under consideration.

We may use the grating in a number of different ways, depending upon the position of the photographic plate, and of the source of light in reference to the axis of the grating. One position is by far the best for photographic work, and in this note the formulas for that case alone are given. This position is that in which the centre of the photographic plate is on the axis of the grating.

To investigate this case fully, return to the general equation, (3). For a fixed value of ν all the spectra are brought to a focus on the curve,

$$r = \frac{\rho\cos^2\mu}{\cos\mu + \cos\nu}$$

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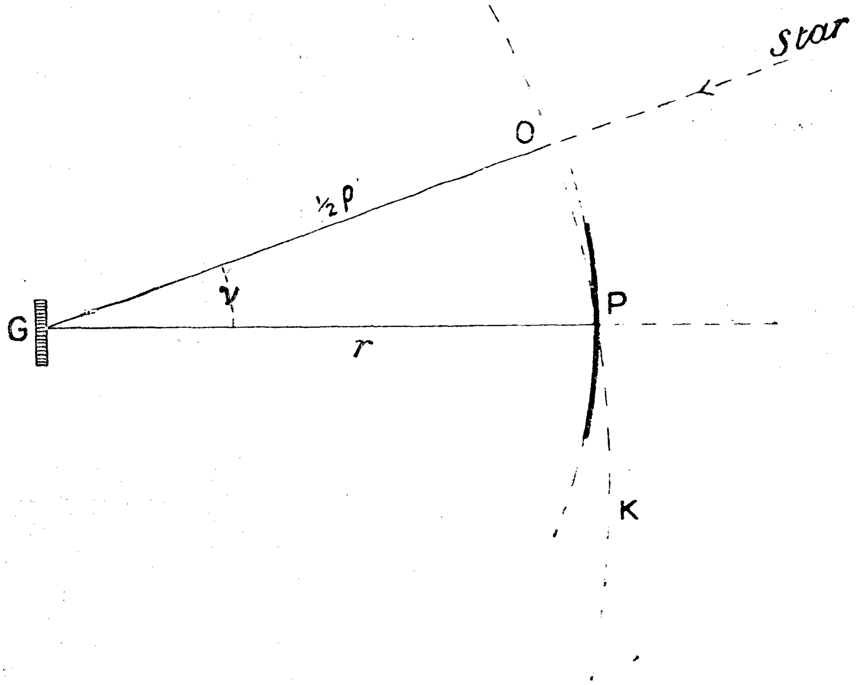
in which cos ν is constant. When μ is so small that its cosine may be taken as unity, this equation reduces to

$$r_0 = \frac{\rho}{1 + \cos \nu} = \text{a constant},$$

and the focal curve becomes a circle. This case is shown in the following figure, in which G is the grating and P the photographic plate.



The light from the star comes in in the direction of OG. The curve, OPK, is a parabola, OG being the half-parameter and equal to $\frac{1}{2}\rho$. For a constant value of ν, those spectral lines on the photographic plate, for which cos μ can be assumed equal to unity, are brought to a focus on a circle whose radius is r. When the dispersion is large, so that cos μ must be taken into account, the focal curve is as defined by equation (3), and the photographic plate must be bent along this curve.

By the theory of diffraction (Rayleigh, *Encyc. Brit.*, "Wave Theory of Light")

$$\lambda = \frac{\omega}{N}(\sin \nu + \sin \mu) \qquad (4)$$

where ω is the grating space and N is the order of the spectrum. From this we have at once

$$\frac{d\lambda}{d\mu} = \frac{\omega}{N} \cos \mu$$

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To find the change in wave length as we pass along the focal curve, we have

$$\frac{d\lambda}{ds}=\frac{d\lambda}{d\mu}\cdot\frac{d\mu}{ds}$$

and

$$\frac{d\mu}{ds}=\frac{1}{\sqrt{r^2+\left(\frac{dr}{d\mu}\right)^2}}.$$

Differentiating the equation of the focal curve, we find

$$\frac{dr}{d\mu}=\frac{r\sin\mu-\rho\sin 2\mu}{\cos\mu+\cos\nu}\equiv\phi(\mu) \tag{6}$$

Whence substituting, we finally find

$$\frac{d\lambda}{ds}=\frac{\omega}{N}\cdot\frac{\cos\mu}{\sqrt{r^2+[\phi(\mu)]^2}} \tag{7}$$

and this is the general formula for the change in wave length along the focal curve.

If, in this equation, we put $\mu=0$, it reduces to—

$$\frac{d\lambda}{ds}=\frac{\omega}{N}\cdot\frac{1}{r_0} \tag{8}$$

a constant. Hence at this point the spectrum is "normal." Within the limits, therefore, in which we can take $\cos\mu$ as equal to unity, the spectrum may be considered as normal.

For a grating of medium dispersion an entire spectrum will be practically normal, provided that the centre of the plate is on the axis of the grating—*i.e.* μ equal to zero for the middle of the spectrum. In the grating used in our experiments the entire first order spectrum subtends an angle of less than 6°. By computation we find from the above formulas that the scales of different portions of the spectrum differ by less than three parts in a thousand. The advantages of this method of working are thus apparent.

In order to test the above method, a small Rowland concave grating with a ruled surface of 1 × 2 inches was used. The grating has a radius of curvature of one metre, and is ruled with 15,000 lines to the inch. The apparatus for mounting the grating is extremely simple, consisting of a light box clamped to the tube of the equatorial—the telescope being used merely as a finder. The light from the star falls directly on the grating, is diffracted and brought to a focus on the photographic plate. The grating is mounted in an ordinary holder, which can be adjusted by side and back screws. The plate-holder holds a plate 1 × 5 inches, bent as closely as possible to the proper curve. The holder is capable of adjustments, so that the plate and the grating can be made parallel. The box is clamped to the telescope in such a way that the lines of the grating are parallel to the equator; and accordingly, by regulating the driving clock of the telescope to run a little too slow or too fast, the spectrum can be made of any convenient width.

For our trials *Sirius* was the star principally used, and the exposures ranged from ten minutes to one hour, according to the width of the spectrum. All the photographs were made with the first order spectrum, and Seed's gilt-edge plates were used. The spectra are about 5 cm. long, and vary in width from 0·1 mm. to 1·5 mm., depending upon the exposure and the rate of the clock. Details of a few specimen plates follow:

Sirius.—November 27. Exposure 40 minutes; width of spectrum, 1·5 mm. Plate shows eight hydrogen and H and K lines.

Capella.—December 9. Exposure 40 minutes; width of spectrum, 0·2 mm. Plate shows F, G, *h*, H, K, and about fifty fine lines.

Procyon.—December 15. Exposure 40 minutes; width of spectrum, 0·15 mm. Plate shows six hydrogen, H, K, and about twenty fine lines.

Rigel.—December 28. Exposure 85 minutes; width of spectrum, 0·1 mm. Plate shows fourteen hydrogen, H, K, and six other lines.

Sirius.—January 3. Exposure 40 minutes; width of spectrum, 0·1 mm. Plate shows sixteen hydrogen, H, K, and fifteen other lines.

The glass positive which accompanies this paper is the enlargement of two of our negatives of the spectrum of *Sirius*. The lower one is the enlargement of a plate taken 1897 December 15; the upper spectrum is the enlargement of the plate taken 1898 January 3 (see above). Since our original negatives are extremely narrow, considerable trouble was experienced in widening out the spectra without introducing spurious lines. Although some of the finer lines in the enlargement are undoubtedly spurious, yet the plate shows the general character of our photographs, in that it shows clearly the hydrogen lines, K, and many other lines which can be easily identified.

All these experiments were carried out in the Observatory, which is most unfavourably situated for such work. We are confident that much better results will be obtained under better conditions, and think that this method promises to become of great value to stellar spectroscopy.

Johns Hopkins University:
1898 *February* 5.

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On a Convenient Method of adjusting a Polar Axis to the Diurnal Motion. By David P. Todd.

[*Communicated by the Secretaries.*]

To eclipse observers often occurs the practical problem of adjusting a polar axis to diurnal motion in the briefest possible time. As a rule their mountings will have no circles, or very small ones; so that the usual method of adjustment either is inapplicable or falls short of the precise alignment desired. Besides this, the finally adjusted axis points in the direction of the true pole, whereas, for best following by clock-motion, it should point toward the apparent or refracted pole, as indicated by Mr. Davidson in his recent paper.

Some years ago, when equipping the U.S. expedition to West Africa for the eclipse of 1889 December 22, I hit upon a method of speedy adjustment which has not, I think, been described heretofore, and which others may find as convenient and expeditious as I have repeatedly since that time. The mechanical requisite is a simple one; the astronomical requisite is that the poleward horizon must be unobstructed—close circumpolars at least must be visible.

It so happened that this latter precluded actual use of this method on the expedition in question; for, obliged as we were to locate in low south latitudes, proximity of a slight southerly eminence cut off our poleward view. But in the expedition to Japan in 1896—provided by the liberality of Mr. D. Willis James, a trustee, and his son Arthur Curtiss James, a graduate of Amherst College—the method was adapted to three equatorial mountings with perfect success. A lesser one is shown in the accompanying figure.

At the upper or left-hand end of the polar axis is shown a small finder, rigidly attached. The axis of it is adjusted parallel to the mechanical axis of rotation of the polar axis in its bearings. This is easily done by collimating on a well-defined object so distant that twice the space between telescope and polar axis is not a considerable part of distance of object itself.

The finder's large field shows an annular reticle, the ring's angular radius being equal to the polar distance of *Polaris*, if the mounting is to be adjusted in the northern hemisphere. Periphery of the ring is graduated counter-clockwise to twenty-four hours. I found by several trials that graduation to subdivisions of five minutes was fine enough. Preparation of those reticles was troublesome and expensive at first, because done with a dividing engine on glass. At last I found they could be made with high accuracy by photography for a few pence each: carefully draft the circle, graduation, and numerals on Bristol board in very fine lines, and reduce photographically, as if a lantern